Interfaces

Gerrymandering for Justice: Redistricting U.S. Liver Allocation

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U.S. organ allocation policy sequesters livers from deceased donors within arbitrary geographic boundaries, frustrating the intent of those who wish to offer the livers to transplant candidates based on medical urgency. We used a zero-one integer program to partition 58 donor service areas into between four and eight sharing districts that minimize the disparity in liver availability among districts. Because the integer program necessarily suppressed clinically significant differences among patients and organs, we tested the optimized district maps with a discrete-event simulation tool that represents liver allocation at a per-person, per-organ level of detail. In April 2014, the liver committee of the Organ Procurement and Transplantation Network (OPTN) decided in a unanimous vote of 22-0-0 to write a policy proposal based on our eight-district and four-district maps. The OPTN board of directors could implement the policy after the proposal and public-comment period. Redistricting liver allocation would save hundreds of lives over the next five years and would attenuate the serious geographic inequity in liver transplant offers.

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Liver allocation systems necessarily recognize geographical limits, because livers from deceased donors cannot tolerate long transport times. Livers are offered to U.S. transplant candidates in decreasing order of medical urgency, as measured by the modeled end-stage liver disease (MELD) score, within 58 geographic units, called donor service areas (DSAs), that are grouped into 11 regions. In 1998, the U.S. Department of Health and Human Services declared its final rule of organ allocation: Neither place of residence nor place of listing shall be a major determinant of access to a transplant (U.S. Department of Health and Human Services 1998). MELD scores range from six to 40; higher scores indicate a higher chance of dying within 90 days if the candidate does not receive a transplant (Freeman et al. 2002). However, a recent analysis showed that a candidate with a MELD score of 38 out of 40 points and who resides in a favorable zip code had an 86 percent chance of receiving a liver transplant in the next 90 days, whereas a candidate with the same MELD score but residing in an unfavorable zip code had an 18 percent chance of receiving a transplant and an 82 percent chance of dying without a transplant in the next 90 days (Massie et al. 2011).

During the 15 years since the final rule was enacted, powerful stakeholders raised political and legal roadblocks that stymied several geographic-equity initiatives. Attempts to reduce geographic disparity engendered such conflict within the liver transplantation community that this period became known as the liver wars. By bringing optimization and simulation together in a novel redistricting approach to the problem, and by faithfully representing the details and specialized concerns of the liver transplant community

within that mathematical program, we designed sharing districts that renewed the community's commitment to equitably treating all candidates.

We exactly solved a zero-one integer program to partition the DSAs into new sharing districts that minimize geographic disparity in liver offers, preserving the DSA boundaries to protect important relationships between donor hospitals and the organ procurement organization (OPO) in each DSA. We evaluated the optimal district plans using a discrete-event simulation of liver allocation that captures clinical details, such as diagnoses, changes in medical status and MELD score over time, and acceptance or rejection of liver offers, to demonstrate that redistricting would save hundreds of lives and restore the principle of liver transplant prioritization for the most medically urgent patients irrespective of their locations. We developed data-driven cost and organ transport models to answer financial and medical objections to redistricting plans. Policy makers recently voted to open our eight- and four-district redistricting proposals to public comment, a necessary step before the policy is finalized.

We organized the remainder of the paper as follows. We describe U.S. liver allocation policy and the geographic inequity associated with the current policy, review related research, and introduce our redistricting integer program. Then, we outline the operation of the liver simulated allocation model and present the results of testing our district plans in this patient-level simulation. Finally, we generalize the lessons of this project for improving healthcare systems with operations research.

Liver Allocation and Geographic Disparity

This paper considers only the allocation of livers from deceased donors in the United States. Living liver donors direct their donations to specific recipients, and living donations comprise no more than a few hundred of approximately 7,000 liver transplants each year.

Each liver transplant candidate is assigned a MELD score, and livers are offered in decreasing order of MELD score to rescue the patients who most urgently need a transplant to survive. Geographic disparity in

liver allocation means that severely ill candidates in some parts of the country die waiting for an organ, or move to another area to be near a different transplant center if they have the means and the knowledge to do so, whereas candidates in other parts of the country who are less urgently in need receive a transplant quickly because of the location of their transplant center.

Each of the 58 local OPOs coordinates the donation process for deceased donors within the geographic boundaries of its DSA. The DSAs are grouped into 11 regions (Figure 1) based on historical relationships between hospitals and transplant centers in the early days of transplantation. In these regions, 133 centers perform liver transplants. The DSAs contain between zero and nine liver transplant centers. Seven of these DSAs do not have a liver transplant center; livers recovered in those DSAs are distributed to transplant candidates elsewhere in the region. The regions contain between six and 18 transplant centers.

Physicians and candidates can and do decline liver offers, because the liver offered is of low quality or for other reasons. An intricate hierarchy of allocation based on clinical factors (blood type, pediatric status of candidate, age of donor, MELD score, and waiting time) dictates the order in which livers are offered. Generally, the liver is offered first to the most medically urgent candidates within the DSA in which the organ was recovered from the donor. If refused by candidates within that DSA, the liver is offered to the most medically urgent candidates in the region. If refused by candidates in the region, the liver is offered nationally.

Livers from deceased donors must be transplanted quickly after being recovered. The cold ischemia time, which is the time that the organ is stored at a low temperature between the organ recovery and the transplantation, must not be too long. The limit on cold ischemia time is not binary; however, the liver's viability and the likelihood of successful transplant decrease as a function of cold ischemia time. Although the livers are rushed to the transplant centers, geographic constraints on allocation are unavoidable. Transporting the organ is only one of the necessary tasks during this narrow time window; therefore, allocating livers without regard to location

Figure 1: The current liver allocation system includes 11 regions, as the shadings represent. The regions reflect historical relationships among early transplant centers and were not designed to achieve any allocation goal(s). Alaska, Hawaii, and Puerto Rico are shown in separate frames in this map and in the maps in Figures 2–5 and Figures 7 and 8.

would be impossible. Organ transports at distances over 1,000 miles are rare.

Deceased-donor liver offer rates and the transplant rates for candidates with the same MELD show wide geographic disparities (Massie et al. 2011, Roberts et al. 2006). Other documented inequities include a 3.3-fold variation in death rates of candidates on the waiting list, a 20-fold variation in transplant rates, and 10-point differences in MELD at transplant for candidates in different DSAs (Yeh et al. 2011). Geographic disparities even cause the observed disparities between liver transplant rates for Caucasians and Hispanics because of the locations in which these populations live (Volk et al. 2009). Some candidates, usually those with more education and higher socioeconomic status, resort to chasing the organ supply by listing at a distant transplant center. Liver transplant candidates had a 20 percent lower risk of death and a 74 percent higher chance of being transplanted if they transferred from their initial-listing DSA to a different one (Dzebisashvili et al. 2013).

Researchers disagree about the fundamental causes of geographic disparities in liver transplantation. Only a small fraction of the people who die in the United States are eligible to be organ donors, and across the DSAs, we see four-fold differences in the death rates of eligible donors (Sheehy et al. 2012). Either prevalence of illness necessitating transplant or differential access to joining the transplant list might cause these differences; the rate of new listings for liver transplants per DSA is closely associated with organ shortage, as measured by higher MELD score at transplant (Yeh et al. 2011). The local OPOs and hospital staffs play a role in medical management of potential donors and obtaining donor consent; therefore, differences among OPOs might lead to more or fewer donations (Ojo et al. 2005).

Liver allocation is contentious because liver transplantation is a prestigious and lucrative undertaking. For more than a decade, transplant centers and congressional delegations in areas that benefit from the current imbalance have strongly opposed changes to liver allocation—disputes characterized as the liver wars (Stolberg 1999).

Washburn et al. (2011) predicted that changing liver allocation by sharing all livers within the existing regions and bypassing the local DSA level of allocation would save 60 lives annually. We have shown,

however, that regional allocation, while saving lives overall, would worsen geographic disparity in the MELD score at which candidates receive transplants (Gentry et al. 2013). That the goals of reducing deaths and offering more equitable access can conflict with each other is puzzling, because if all candidates had equal access to livers then fewer candidates would die while waiting for an organ. The problem is that drastic imbalances between demand and supply prevail even among the larger regional groupings, given the existing suboptimal regions. Sharing within optimal districts would improve both metrics.

The potential of the optimized redistricting method we present in this paper has broken a long-standing stalemate over redressing geographic disparities in liver allocation. A redistricting approach forces members of the transplant community to endorse or reject the goal of minimizing geographic disparity in liver offers, rather than just object to some detail of a particular allocation plan. The constraints and objective of the integer program are transparent and represent a consensus of the liver and intestinal organ transplantation committee of the Organ Procurement and Transplantation Network (OPTN), after that group worked extensively with our team to clarify the requirements of a districting plan and the outcomes to be monitored.

Offering an optimization tool to improve allocation is not a panacea for resolving disputes. In practice, the priorities and constraints might remain contentious, even after the modeling and technical challenges of redistricting are addressed, as in the example of the Philadelphia City Council redistricting contest (Gopalan et al. 2013). Altman (1997) argues that social goals cannot be unambiguously characterized and weighed quantitatively; therefore, optimal redistricting can never be neutrally implemented. Optimization cannot create a helpful veil of ignorance (Altman 1997) about winners and losers in liver redistricting, because the balance of existing inequities is clear to most participants.

Related Research

Cope (1971) uses the term regionalization for the problem of aggregating a finite, denumerable, nonoverlapping set of units into districts in accordance with a set of criteria. Regionalization—usually

called districting or redistricting—appears in many contexts, with a diversity of criteria that might appear in the objective, constraints, or exclusion rules, or be embedded in heuristic district generation procedures. Caro et al. (2004) offer an excellent, detailed, and fairly recent survey of redistricting models.

The canonical redistricting problem is designing political districts in which the important considerations are contiguity and compactness of the districts and strict constraints that districts must contain nearly equal populations (Murphy et al. 2013). Although competing definitions of compactness abound, some compactness metrics have long been incorporated straightforwardly into linear or integer programs (Garfinkel and Nemhauser 1970). In contrast, although districts can be unambiguously classified as contiguous or noncontiguous, researchers only recently found computationally tractable representations for contiguity constraints (Shirabe 2009).

In the obvious formulation as a set-partitioning optimization, computational hurdles to exact solutions proved severe for most applications. Garfinkel and Nemhauser (1970) used an implicit enumeration scheme for calculating exact solutions and found the largest solvable instances had about 50 population units. Mehrotra et al. (1998) applied a branch-and-price graph-partitioning methodology to partition 46 counties into six districts in South Carolina. Birge (1983) formulated a quadratic program and heuristic solution procedure to partition 83 counties into 38 districts for the Michigan Senate. To resolve 35,000 meshblocks into 95 districts, George et al. (1997) used a heuristic location-allocation algorithm adapted from Hess et al. (1965), which iteratively refined the district centers. Our liver redistricting model employs the location-allocation constraints suggested in Daskin (2010).

In assigning students to school districts, the focus is usually on minimizing travel distance or busing, while respecting school capacity limits and sometimes desegregation mandates. In this problem class, computationally tractable algorithms do not always guarantee contiguity and compactness of districts. In Liggett (1973), the authors applied an implicit enumeration method to show that desegregating Pasadena schools did not require increasing the number of bused students. A heuristic and interactive

approach to minimizing busing appeared in Ferland and Guénette (1990). Caro et al. (2004) provided a comprehensive tool that minimizes travel while enforcing capacity constraints at each grade level, racial-balance constraints, stability constraints requiring a degree of similarity to the current districts, contiguity, and compactness. The resulting zero-one integer program can usually be solved quickly, and solutions can be interactively adjusted and reoptimized.

An early description of an optimal districting model in healthcare, in which balancing demand and provision of healthcare services is a constraint, appeared in Ghiggi et al. (1976). Blais et al. (2003) incorporated travel time on Montreal transit services as a component of workload in designing equal-workload districts for home healthcare workers with a Tabu search heuristic. Using adjacency-tree representation to generate only contiguous districts, Zoltners and Sinha (1983) surveyed several contrasting objectives in designing sales territories: minimizing disruption from an existing sales territory map, minimizing travel, and equalizing workload or sales potential.

Prior Work on Redistricting Liver Allocation

Other researchers have explored redistricting models for liver allocation; however, these models have had significant limitations. First, previous studies either defined geographic disparity with an implausible metric or disregarded geographic disparity. Second, previous studies defined efficiency by making a strong but medically inaccurate assumption about the functional relationship between liver viability and transport distance. Third, previous studies employed a notion of recipient-donor matching that does not exist in liver transplantation, resulting in districts optimized to an artifact of the input data. Fourth, previous studies neglected to model accept or decline decisions that shape liver allocation and did not consider the modern (2002 to present) MELD-score prioritization of liver transplant candidates.

Disregarding geographic equity, Kong et al. (2010) redistricted liver allocation to maximize an efficiency metric of the number of successful transplants. Kong et al. (2010) used a branch-and-price algorithm combined with a geographic-cover decomposition scheme to approximate solutions to the set-partitioning problem. Stahl et al. (2005) redistricted liver allocation

with an objective that balanced efficiency with geographic equity as measured by intradistrict transplant rates, but were forced to use nine or fewer DSAs per district because of computational challenges. Demirci et al. (2012) applied geographic covers with a branch-and-price approach to explore the efficient frontier in a trade-off between efficiency and geographic equity, allowing them to consider many districts that Stahl et al. (2005) excluded.

The geographic-equity objective in both Stahl et al. (2005) and Demirci et al. (2012) maximized the minimum in-district viability-adjusted transplant rates per waiting list candidate. The range of transplant rates, however, is not a credible metric of geographic disparity. The transplant rates per candidate, neglecting MELD-score prioritization among waiting list candidates, are sensitive to differences in local waiting list patterns that might distort the number of waiting list candidates. We discuss the problem of sensitivity to local waiting list patterns in the *Input Parameters for the Redistricting Zero-One Integer Program* section. In contrast to these efforts, we optimize a pure geographic-disparity metric based on the balance of organs supplied and organs demanded by patients who have priorities high enough to warrant transplants. Because geographic disparity causes excess waiting list deaths, we simultaneously achieve improved allocation efficiency, as measured by a lower number of patients who die while on a waiting list.

In creating a viability adjustment to penalize longer organ transports, previous studies assumed that a liver's viability is a closed-form function of the cold ischemia time and assumed that cold ischemia time is a closed-form function of the organ transport distance. The clinical transplant literature does not include support for describing liver viability as a function of the transport distance, because cold ischemia time varies widely, almost irrespective of the transport distance (Gentry et al. 2014). Moreover, these studies assumed that patients accept or decline liver offers based only on the organ transport distance. The studies also treated livers as wasted if they were declined once; however, livers are not recovered until they are accepted. When a liver offer is declined because the transport distance is too long or for other reasons, the liver is not wasted; it is offered to another

patient. The clock does not start ticking until after a patient and his (her) transplant team have decided that the transport time for the liver is acceptable. In our study, we handle distance by setting an upper bound on transport time within a district as a feasibility constraint.

Prior work defined efficiency as a count of the number of intradistrict liver transplants, balancing the purported lower likelihood of finding a matched recipient in smaller districts against the lower likelihood of a successful liver transplant with longer transport distances in larger districts. An affinity value was calculated between every pair of DSAs, representing the likelihood of donated livers flowing from DSA i to DSA j in a geography-independent allocation scheme. The affinity value was variously based on wait list population sizes (Stahl et al. 2005) or on a pre-MELD-era simulation (Shechter et al. 2005) of liver allocation (Kong et al. 2010). Combining the affinity values with a quantity representing the fraction of organs from each DSA that were expected to be allocated at a national level, these studies estimated the number of intradistrict transplants for various potential districts. This form of penalty for smaller districts reflects an assumption that livers leave the districts and reach a national level of distribution because no suitably matched recipient resides within the district. But, unlike kidney transplants, liver transplants do not require careful matching between donor and recipient. The percentage of livers that flow from one DSA to another in simulation and in actual situations depends on the timing of the donations, the evolution of medical urgency among transplant candidates, the allocation hierarchy, and the decisions to accept or decline organs. Donors in DSA i cannot be more compatible with the patients in DSA j than with the patients in DSA k.

This efficiency metric also treats livers not accepted within a district as if they were discarded; in reality, livers not accepted within a district are often transplanted at a national level. Livers reach a national level of distribution because the livers are higher risk or lower quality than most (Lai et al. 2012), not because of compatibility issues. A handful of aggressive transplant centers perform most transplants with nationally distributed livers (Garonzik-Wang et al. 2013). Because the simulation tool in

Shechter et al. (2005) assumed all offers are accepted, none of these dynamics would have been present in the input data for redistricting in the studies we have been discussing. In reality, a high or low intradistrict transplant rate would reflect the distribution of aggressive centers across the districts. Some livers are destined to aggressive centers regardless of the arrangement of the districts; therefore, zeroing out the value of a liver donated in DSA i and transplanted in DSA j because the district plan puts those two DSAs in different districts would not make sense.

Finally, any redistricting optimization study must simplify or remove many medical details to reach a stylized mathematical programming representation of liver allocation. The challenge is then to answer transplant stakeholders' questions about the impacts of redistricting, when many clinical details are not explicitly retained in the optimization model. Kong et al. (2010) and Demirci et al. (2012) tested their optimized district plans using a patient-level allocation simulation; however, neither the simulation nor the mathematical formulation incorporated MELD scores, the current mainstay for prioritizing patients, or modeled decisions to accept or decline offers. To answer questions about the clinical impacts of redistricting, we tested our optimal district plans in a sophisticated, validated patient-level simulation that handles MELD scores, models accept or decline decisions, and that the transplant community accepts. We also carried out an extensive patient-level cost analysis to predict the financial impacts of redistricting.

Optimal Redistricting for Liver Allocation

The Department of Health and Human Services oversees transplantation; the OPTN is responsible for developing organ allocation policies through its several committees (McDiarmid et al. 2008). We worked closely with the liver and intestinal organ transplantation committee over three years to develop the redistricting model we present in this paper. The primary goals of redistricting were to reduce geographic disparity in liver allocation and to reduce the number of deaths of patients on the liver transplant waiting list without imposing an unacceptable organ transport burden.

Our redistricting integer program partitions the set of DSAs into a fixed number of districts to minimize the sum of absolute differences between the number of deceased-donor livers recovered in each district and the ideal number of livers that would be offered in each district if each liver was given to the most medically urgent candidate in the country. That is, we minimize the number of livers misdirected away from the most medically urgent candidates. The liver and intestinal organ transplantation committee chose not to revise the DSA boundaries, because each DSA has an OPO with staff who have long-standing relationships with the donor hospitals in the DSA. By contrast, the 11 regions are administrative groupings that could be rearranged without major upheaval.

The formulation includes location-allocation constraints that select one DSA as the center of each district and assign each DSA to the district center closest to it to produce compact regions. The districts we design are essentially contiguous, although our formulation does not guarantee this. Where our districts depart from contiguity, it is because the DSA boundaries are not uniformly contiguous (Figure 2). Several DSAs are the union of noncontiguous territories; the division of Texas into its three DSAs is a striking example.

The liver committee approved a handful of constraints on the district plans. The committee members asked to examine the plans for districts with between four and eight districts; therefore, we solved the redistricting program with numbers of districts fixed at four, five, six, seven, and eight. As a side constraint on this zero-one integer program, the committee set an upper bound on the transport time between the center of each district and each DSA included in the district. The committee asked to examine plans in which this upper bound was set at four hours and at five hours, but our experiments found that an upper bound of three hours does not harm the other outcomes and yields districts that are more broadly acceptable. The committee required that the expected number of waiting list deaths must either decrease or remain constant. Waiting list deaths are not explicitly represented in the integer program; in the *Liver Simulated Allocation Model and Results* section, we describe how these can be estimated with the liver simulated allocation model (LSAM), given any particular district plan.

Later in the process, the committee added a new constraint—that every district must contain at least six transplant centers. This requirement became clear when members objected to a provisional district map because a district contained only two transplant centers. Deceased-donor livers vary in quality, and marginal livers are more likely to be used and less likely to be discarded when more competition exists among transplant centers (Halldorson et al. 2013, Garonzik-Wang et al. 2013). Some people believe that the current lack of competition in DSAs with only one transplant center causes some OPOs to not pursue some donors whose livers would have been transplantable. Unlike voting districts, liver allocation districts need not be of equal size or population; rather, the goal is that districts achieve a similar balance of supply and demand. The lower bound on number of transplant centers, however, effectively places a lower bound on the size of the districts.

Throughout the process, members of the committee sought to understand the trade-off between the fairness of the system and the distance the livers would have to travel. Still, we were forced to abandon a formulation of the objective that used a linear combination of fair and far metrics, because the weighting parameter lacked meaningful units, which made the redistricting procedure opaque. Transparency is essential when making policy regarding a collective resource such as deceased-donor organs; as a result, all the numerical parameters in our final zero-one integer program are concretely defined quantities familiar to transplant stakeholders.

An earlier version of our LSAM (Gentry et al. 2013) minimized a sum of the squared-distances compactness metric, with geographic fairness appearing as a constraint. Hess et al. (1965) used a population-weighted version of this compactness metric in a political-redistricting problem. Minimizing the sum of squared distances does not guarantee contiguous districts. In our application, minimizing the sum of squared distances yielded districts with an unacceptably high degree of noncontiguity. Another difficulty with this formulation is selecting and (or) iteratively refining the fixed district centers, because the solutions are sensitive to this basically arbitrary selection.

We reaped the benefit of 40-plus years of hardware and software development and were able to solve our

Figure 2: Of the 58 local DSAs in the United States, several are defined as unions of noncontiguous geographic blocks.

small 58-unit districting problems in about an hour on a personal computer using the general mathematical programming solver OPL/CPLEX, version 12.1.

Input Parameters for the Redistricting Zero-One Integer Program

The supply of deceased-donor livers for each DSA can be defined as the number of livers transplanted from adult donors in the DSA in 2010.

The demand for liver transplantation in each DSA is difficult to define for several reasons. The population on the liver waiting list in each DSA is dynamic. New transplant candidates arrive, and some candidates are removed because they have died, become too sick to transplant, or recovered without a transplant. Also, each candidate's MELD score fluctuates over time as his (her) disease advances or condition improves, or medical crises occur. We approximate the demand for liver transplantation by capturing all incident (arriving) adult candidates in each DSA during 2010 and recording their highest MELD scores as of December 31, 2010. Notionally, we distribute the livers donated in 2010 to this static pool of candidates in decreasing order of MELD score until the supply is exhausted, and we count the number of livers that would have been allocated to each DSA under this

distribution system as the ideal number of transplants for that DSA.

We considered other methods for defining the transplant demand in each DSA. For example, we could use the raw counts of new waiting list candidates in 2010; however, that method would be sensitive to the number of low-MELD-score candidates added. Because candidates with a MELD score below 15 do not derive a survival benefit from transplantation, and because some candidates with even moderate MELD scores stand little chance of receiving a liver offer in some parts of the country, transplant centers differ in their practices of listing candidates with low MELD scores. The method we chose has the advantage of being insensitive to local handling of such candidates.

Because transport delays can extend the cold ischemia time, thus impacting the likelihood of a successful transplant, we created an intricate geographic information system (GIS) model for liver transport time (Gentry et al. 2014). Livers are transported by car, by chartered plane, or occasionally by helicopter. We located hospital street addresses, and for every donor hospital and transplant center pair, we calculated via Google the driving time between the hospitals directly and to each hospital's nearest airport.

We created a separate model to estimate flying time between airport pairs. We also added logic to select driving or flying as the transport mode and thereby estimated a transport time between each donor hospital and transplant center pair. Finally, we aggregated these times to compute a transplant-volume-weighted average transport delay between each pair of DSAs, as we outline in the appendix.

Liver Simulated Allocation Model and Results

Although we solved the LSAM program exactly to obtain district plans similar to those in Figures 3 and 4, our mathematical programming model (see the appendix) reduces the true dynamic, medically complex system to a static and uniform quantification. Transplant care providers, well acquainted with the temporal evolution of liver disease and with the nuanced judgment required to decide whether a particular organ would benefit one of their patients, are unlikely to trust in such an obviously truncated representation of liver allocation.

The LSAM is a discrete-event simulator that captures dynamic patient-level detail and facilitates comparison of proposed liver allocation rules (Thompson et al. 2004). Over the past decade, policy makers

have relied heavily on the LSAM to project outcomes of allocation changes and inform decisions. Another team of researchers initially constructed the LSAM, but we worked with our colleagues at the Scientific Registry for Transplant Recipients to enhance the tool so it could answer redistricting questions. In particular, we added a resampling module to extend the simulation from a one-year to a five-year projection and integrated our transport-time estimates.

Figure 5 depicts the operations of the LSAM. We simulate liver allocation from January 1, 2006 to December 31, 2010, starting with the patients on the waiting list on January 1, 2006 and resampling patient arrivals to the waiting list thereafter. We model the disease progression of each patient on the waiting list by resampling every MELD score update independently from the MELD history of a similar patient, matched on clinical characteristics. When a liver arrives, it is offered to the highest-priority patient according to the allocation scheme being simulated. We model the probability of acceptance for each offer using a logistic regression based on the organ quality, the patient's medical history, and the transport distance. If a patient declines an offer, the organ is offered to the next-highest-priority patient. Simulated patients might die on the waiting list, be removed from the waiting list

Figure 3: Minimizing disparity in liver allocation using four organ-sharing districts yields these districts.

Figure 4: Minimizing disparity in liver allocation using eight organ-sharing districts yields these districts.

for other reasons, die after receiving a transplant, or have a transplant fail and go back on the waiting list.

The LSAM provides detailed patient-level output, including deaths of candidates on the waiting list, deaths of patients after transplant, liver discards,

transport distances and times, and the MELD scores at which patients are transplanted. Only this type of simulation could have predicted the age distribution and racial and ethnic composition of the transplanted population, the post-transplant death and relisting rates,
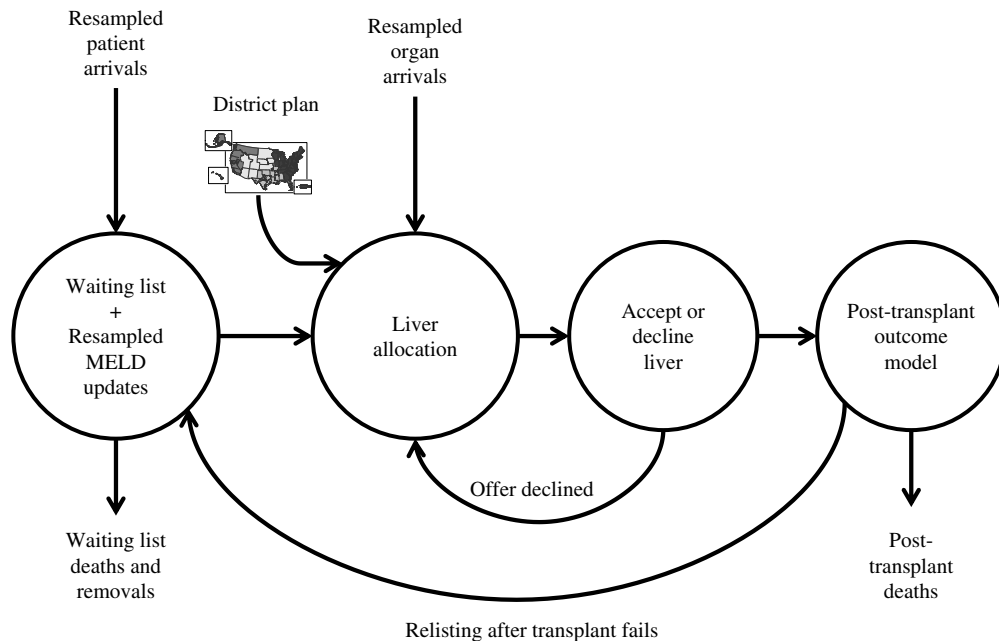

Figure 5: Policy makers use the LSAM to compare alternative organ allocation schemes.

and the pattern of organ flow across local boundaries under different allocation schemes. All these predictions are essential for policy makers.

Over five years, we ran 10 simulations using the LSAM for each allocation scheme tested. As a variance-reduction technique, we used the same set of 10 resampled input files and the same random-number-generator seed for each allocation scheme tested. Using the sign test, we performed pairwise comparisons between output metrics for significance at the $p < 0.05$ level, comparing the current allocation scheme with each of the other schemes.

Local allocation means livers are offered to the most medically urgent candidate in the DSA. Regional allocation means livers are offered to the most medically urgent candidate in the regions defined by the current 11-region map in Figure 1. National allocation means all livers are offered to the most medically urgent candidate anywhere in the country, regardless of transport time or distance. Circular allocation means each liver is offered to the most medically urgent candidate

within 500 miles of the donor hospital. In all redistricting plans, livers are offered to the most medically urgent candidate in the district.

In Table 1, we report formative redistricting results concerning the trade-offs among waiting list deaths and total deaths, the upper bound for transport time, and two metrics of geographic disparity. The zero-one integer program minimizes the number of misdirected livers, that is, the sum of absolute differences between the ideal number of livers and actual number of livers transplanted in each district. The liver committee, however, has focused on a different metric of geographic disparity—the standard deviation of median MELD score at transplant across DSAs. MELD score at transplant is a natural and routine metric for transplant stakeholders, unlike our notion of misdirected livers. Moreover, the explicit organizing principle of liver allocation is to offer the liver to the most medically urgent candidate. The standard deviation of median MELD at transplant across DSAs summarizes the variability in medical urgency for

Allocation	Districts	Transport (hrs)	Misdirected	Std dev MELD	Net wait deaths	Net deaths
Local	11	—	2,363	3.01	0	0
Regional	11	—	1,317	3.26	−165	−122
National	1	—	0	1.66	−344	−510
Circular	—	500 miles	—	2.70	−244	−112
Redistrict	4	2.5	Infeasible	Infeasible	Infeasible	Infeasible
Map, Figure 3	**4**	**3**	**128**	**1.87**	**−554**	**−581**
Redistrict	4	4	50	2.11	−502	−520
Redistrict	5	2.5	Infeasible	Infeasible	Infeasible	Infeasible
Redistrict	5	3	129	2.01	−468	−442
Redistrict	5	4	76	2.09	−479	−539
Redistrict	5	5	49	2.05	−461	−466
Redistrict	6	2.5	Infeasible	Infeasible	Infeasible	Infeasible
Redistrict	6	3	136	2.12	−433	−465
Redistrict	6	4	88	2.22	−384	−353
Redistrict	6	5	62	2.17	−422	−520
Redistrict	7	2.5	407	2.76	−379	−376
Redistrict	7	3	149	2.04	−374	−382
Redistrict	7	4	97	2.24	−358	−387
Redistrict	8	2.5	544	2.78	−325	−304
Map, Figure 4	**8**	**3**	**156**	**2.08**	**−332**	**−342**
Redistrict	8	4	109	2.24	−357	−375
Redistrict	11	3	276	2.44	−211	−240

Table 1: The liver committee's primary charge was to reduce disparity (i.e., to reduce the standard deviation of median MELD score at transplant); however, limiting transport time and reducing the number of deaths were also important.
Note. Boldface rows correspond to the district maps shown in Figures 3 and 4, which were released in the liver committee's concept document on redistricting.

candidates who reach the priority needed to receive a liver transplant in different parts of the country. More fair and equitable distribution of livers is associated with a lower standard deviation of the median MELD score.

All quantities in the last three rows of Table 1 are statistically significantly different ($p < 0.05$) from the values for the current allocation system (first row) based on the sign test. In local allocation, livers are offered to the most medically urgent candidates in the DSA. Regional allocation uses the existing 11 regions; however, livers are offered to the most medically urgent candidates in the region. National allocation means all livers are offered to the most medically urgent candidates in the country. Circular allocation means each liver is offered to the most medically urgent candidate within 500 miles of the donor hospital. In all the redistricted plans, livers go the most medically urgent candidates in the district. The table includes two factors that were design parameters for redistricted maps: the number of districts and the limit on transport time from any DSA to the district center. The redistricting objective was misdirected livers, which is the sum of absolute differences between the actual and ideal number of livers available in each district. The standard deviation of the median MELD score at transplant across DSAs is another metric of geographic disparity. Net wait deaths is the average difference between the number of deaths of patients on the waiting list in the allocation scheme being tested and the number of deaths of patients on the waiting list in local allocation. Net deaths is the average difference between the total number of deaths in the allocation scheme being tested and the total number of deaths in local allocation.

Table 1 demonstrates some discrepancies between these two metrics of geographic disparity. In a national system, the number of misdirected livers is zero per our definition, which treats all transplant candidates as interchangeable and treats all livers as simultaneously offered and uniformly accepted. Discrepancies exist between misdirected livers and the MELD-score measure of inequity for several reasons: (1) livers can only be offered to candidates who are waiting when the liver becomes available, (2) livers are offered according to blood type and other clinical criteria, and (3) liver offers are sometimes declined.

For example, even in a national distribution system, the LSAM predicts some variability in the median MELD score at transplant per DSA; thus, 1.66 is close to the lowest achievable value for this measure of geographic disparity.

We asserted that regional allocation with the existing 11 regions would worsen geographic disparity compared with local allocation. Comparing the first two rows of Table 1, the standard deviation of median MELD score increases from 3.01 for local allocation to 3.26 for regional allocation. To make clear whether this difference should be seen as significant, for each district plan, Figure 6 shows the range of measured standard deviation of median MELD score at transplant over 10 simulation replications. Regional allocation is demonstrably less equitable than local allocation, and both local and regional allocation are significantly less equitable than any of the redistricted alternatives on this metric.

For any fixed number of districts, the objective value to be minimized (i.e., the number of misdirected livers) decreases as we relax the constraint on transport time. Our estimates of the standard devi-
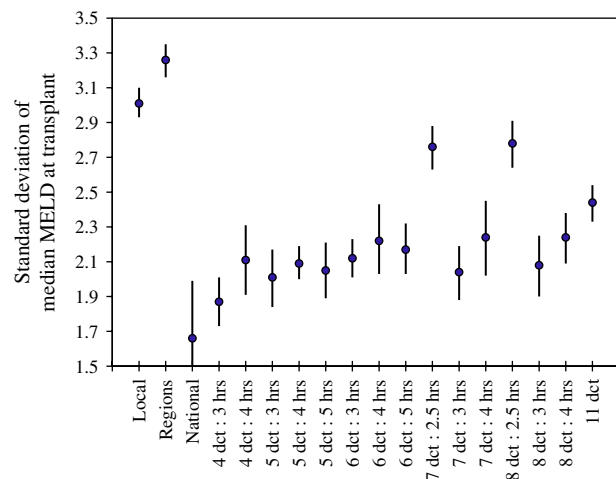

Figure 6: (Color online) For district plans with varying numbers of districts and varying upper bounds on transport time, the range of values over 10 simulations of the sample standard deviation of median MELD score at transplant overlaps to a significant extent for redistricted plans. The exceptions are the district plans with a 2.5-hour transport limit, which appears to be too short to provide equitable allocation.
Note. **Abbreviation: dct represents number of districts. These infeasible combinations are not shown: four, five, and six districts with 2.5-hour transport-time limit.**

ation of median MELD at transplant, however, do not strictly decrease as we relax the constraint on transport time. Figure 6 shows that the small differences in Table 1 between the measured standard deviations of MELD are mostly within the range of simulation error. All district plans with between four and eight districts yield roughly the same reduction in geographic disparity, except the district plans under the strictest 2.5-hour transport-time constraint, which seems to be too tight a bound to allow geographically equitable distribution of livers. A strong trade-off between resolving geographic inequity and relaxing the transport-time constraint or increasing the number of districts is not apparent in Figure 6, although the figure shows what might be a slight trend toward higher MELD-score variation with more districts.

In contrast, Table 1 does indicate a trade-off between the number of districts and the number of patients on the waiting list and number of total deaths. We could explain this by better timing; in fewer larger districts, a critically ill candidate might be more likely to receive an offer in time to save the candidate's life.

One alternative allocation scheme that has generated interest is circular allocation, which gives priority to candidates within a fixed distance of each donor hospital. Both heart and lung allocation proceed from local DSAs to concentric-circle geographic zones (Colvin-Adams et al. 2012). The advantages of circular allocation are simplicity and transparency; however, circular allocation is an ad hoc approach and

should be expected to perform poorly relative to optimal districts, at least with respect to the metric being optimized. Circular allocation with a 500-mile radius would require about the same organ transport distance and time as regional allocation with 11 districts. Indeed, compared to the existing 11 regions, circular allocation is fairer; however, compared to any of the optimized district systems, circular allocation is less fair, as measured by standard deviation of median MELD score at transplant. In Table 1, we have also included an example of an optimal district plan with 11 districts for comparison with the existing 11-region system, although policy makers focused on options having between four and eight districts.

The maps in Figures 7 and 8 offer another illustration of reduced geographic disparity under redistricting. These maps show areas in which the median MELD at transplant is very low or very high as the lightest and darkest areas, respectively. Areas marked as none are DSAs without liver transplant centers. Many transplant centers have either very abundant access (lightest color) or very poor access (darkest color) to liver offers; redistricting would give most areas of the country similar access to liver offers.

Figures 7 and 8 also show that these redistricting plans would not bring the MELD score at transplant for California transplant centers in line with the rest of the country. Under any feasible district plan, the district containing California will have the lowest ratio

Figure 7: Compared to current local allocation (left), a redistricting plan with four districts (right) markedly reduces geographic disparity in the median MELD score at which candidates are offered transplants.

Figure 8: Compared to current local allocation (left), a redistricting plan with eight districts (right) markedly reduces geographic disparity in the median MELD score at which candidates are offered transplants.

of donors to new liver transplant candidates (another rough metric of liver availability). That is, California's transplant candidates might be so disadvantaged by California's geographic isolation from areas with higher donor availability that even candidates in optimal districts will have insufficient access to liver offers.

Addressing Other Concerns of Transplant Stakeholders

Changing the geographic hierarchy of liver allocation would have a huge impact on transplant stakeholders. A proposed policy change is likely to be shelved if decision makers find the impacts on transport time, minority and pediatric populations, or especially costs, to be unpalatable. These aspects of the redistricting problem are not primary; we do not explicitly represent them in the objective or constraints of the zero-one integer program, except for the transport-time bound. To explicitly represent every quantity of interest in liver allocation in a mathematical program is almost impossible, because the problem size would explode at this level of detail. Rather, we design the most equitable district plan that is feasible, given a limited set of constraints, and subsequently evaluate other impacts.

Table 2 compares the median transport time, median distance, percentage of organ transports that require flying, total organ transport cost, and total cost for the primary redistricting alternatives. We

used the LSAM's patient-level output data to estimate the costs for transporting organs and for pretransplant, transplant, and post-transplant care. Adapting from Salvalaggio et al. (2011) and Axelrod et al. (2014) regression models with spline terms that were built using historical cost data, we demonstrated that redistricting would be either cost neutral or cost saving compared with the existing allocation. Although our four- and eight-district plans would substantially increase the cost of transporting livers, these plans would decrease costs because severely ill patients would get transplanted more quickly instead of remaining in the hospital over extended periods of time. The costs of transporting organs are ultimately covered by the charges for transplantation.

Allocation	Median time (hours)	Median distance (miles)	Flying (%)	Transport cost (millions) ($)	Total cost (millions) ($)
Local	1.7	122	53	298	9,268
Regional	1.9	194	66	329	9,150
Four districts (Figure 3)	2.3	419	84	467	9,282
Eight districts (Figure 4)	2.0	243	73	383	9,145

Table 2: Redistricting would minimally extend transport times and would be either cost neutral or cost saving, because the increased organ transport costs are offset by reductions in patient-care costs. The columns show median transport time, median distance, percentage of organ transports that require flying, total organ transport cost, and total organ transport plus patient-care cost.

Transplanted recipients might (or might not) experience cost changes, because private or public insurance generally covers liver transplantation. However, even revenue-neutral cost shifts would be disruptive for transplant centers, hospitals, and OPOs, particularly because the pretransplant care costs are not generally borne by transplant centers. Gentry et al. (2015) includes a full report of the cost model.

Geographic disparities in liver transplantation have also negatively affected Hispanic patients, because the Hispanic population is not uniformly distributed throughout the country. Another advantage of redistricting is that it would cause a statistically significant increase in the percentage of transplants offered to Hispanic candidates—from 14.0 to 14.7 percent under the four-district plan in Figure 3; the Minority Affairs Committee welcomed this news. Pediatric candidates would comprise a higher percentage of transplants under redistricting, rising from 7.1 to 7.7 percent with the eight-district plan in Figure 4 or 8.5 percent with the four-district plan in Figure 3. This is encouraging news, because the allocation system explicitly favors pediatric candidates.

Although the persistent geographic inequity in liver allocation is universally acknowledged, no uncontroversial metric exists to represent the unequal access that candidates have to liver offers. Lest this ambiguity be a barrier to resolving the problem, at its November 2012 meeting, the OPTN board of directors directed every organ-specific committee to define a primary metric of geographic disparity in allocation. In our zero-one integer program, we minimized a quantity we call misdirected livers; however, the liver committee used the standard deviation of median MELD score at transplant to evaluate the district plans. Other measures of geographic disparity could be proposed, each with its own drawbacks. For example, the variability in waiting list death rates per DSA is a direct measure of the primary outcome for patients; however, the number of deaths in each DSA is too small to be significant in measuring differences between allocation schemes, and this metric might also be influenced by varying local decisions about whether and when to add low-priority candidates to the waiting list.

One remaining concern for some stakeholders is whether a fixed-district plan will perform well over time. District boundaries might need adjustments as populations shift and transplant centers open and close. In particular, Puerto Rico's first liver transplant program opened in late 2012; therefore, we did not account for it in the district plans we present in this paper. A recently available drug that cures Hepatitis C might change the balance of supply and demand, because Hepatitis C is a frequent indication for liver transplant. Most of these developments will move slowly, and we do not anticipate that the district boundaries would be revised more often than once per decade. We have conducted sensitivity analyses (Gentry et al. 2013) demonstrating that districts designed using 2006 input parameters performed extremely well in simulations using 2010 data.

The next frontier for operations research in transplant policy making is building game-theoretic behavior models that predict how changes in organ allocation rules are likely to change decisions to accept or decline organ offers. At present, this is a weakness of the LSAM and other organ allocation simulators; however, success in building responsive acceptance models to replace simple logistic regression on historical data will not come easily. The available data on offers and refusals defy explanation and do not seem to fit any rational-choice assumption (Alagoz et al. 2007). Although physicians must enter a refusal reason when an organ is declined, the reasons given are sometimes inconsistent with the clinical outline of the case.

Conclusion

On April 1, 2014, the liver committee decided in a unanimous vote of 22-0-0 to prepare a policy proposal based on the four-district plan (Figure 3) and the eight-district plan (Figure 4). Following the June release of a concept document about redistricting, in September 2014, the OPTN convened a public forum on redistricting, which was its most highly attended forum in history. After considering revisions based on discussions at the forum and following a required public-comment period, the policy could be implemented with the approval of the OPTN board of directors. By creating a transparent, quantitative optimization model for liver redistricting, we achieved an unprecedented consensus on a policy that promises to save lives and make liver allocation more equitable.

Why was this redistricting project transformative; why did it bring the liver community to a new resolution to reduce geographic disparities? One reason might be the clarity of optimizing a concretely stated objective to break the cycle of opponents repeatedly challenging any proposed plan by proposing a slightly different plan. The sustained participation of many transplant professionals in constructing the objective, constraints, and evaluation metrics undoubtedly generated confidence in the feasibility and effectiveness of the district plans. A redistricting model would not have been persuasive to the transplant audience if we had not explicitly engaged the complex medical, ethical, historical, and practical dimensions of the problem. Validating our district plans with a patient-level simulation was indispensable.

We likely had more credibility as partners in using operations research to support transplantation because our team had been influential in optimizing kidney paired donation (Segev et al. 2005, Gentry et al. 2007, 2009). Our simulations quantifying the benefits of kidney paired donation helped Congress pass the necessary legislation (U.S. Congress 2007). We donated algorithms, software, and research time to the OPTN as it established a U.S. registry to facilitate kidney exchanges between transplant candidates whose intended living donors were incompatible.

These lessons can be taken as generalizable rules for using operations research to make a meaningful impact in healthcare: when the mathematical formulation of a healthcare problem must be significantly simplified, answer providers' clinical questions with medically realistic simulations or a trial application of your solution; commit to a sustained and deliberate relationship with the healthcare providers and administrators involved, taking the time to fully explore goals and feasibility constraints; and emphasize the advantages of a quantitative approach to designing systems that better serve patients and providers.

Appendix. Zero-One Integer Programming Formulation of the Liver Redistricting Model

The set $\mathcal{I}$ consisting of mutually exclusive, geographically defined donor service areas (DSAs) i is to be partitioned into N districts. We use a location-allocation formulation as presented in Daskin (2010) to create (usually) contiguous districts for liver sharing. We choose one DSA as the center of each of the N districts. Binary decision variables Y_k are 1 if DSA k is assigned as the center of a district, and 0 if it is not assigned. Another set of binary decision variables, W_{ik}, are 1 if DSA i is in the district with center at DSA k, and 0 if not. For clarity, we also use the notation $\mathcal{K} \equiv \mathcal{I}$ for the set of DSAs, wherever an element $k \in \mathcal{K}$ should be taken as a candidate center of a district.

Let the number of livers recovered for transplantation in DSA i be d_i (during a specific time window). We define the ideal or perfect number of donors p_i for each DSA i as the number of transplants that would have occurred in DSA i if all $\sum_{i \in \mathcal{I}} d_i$ livers were distributed in order of decreasing MELD score to all candidates who joined the waiting list during that time window, assuming all candidates accept any offers. Let h_i be the number of transplant centers in DSA i, and let $\underline{h}$ be the lower bound on the number of transplant centers per district.

We calculate δ_{ij} as the mean of the transplant-volume-weighted distance between donor hospitals in DSA i and transplant center in DSA j, and the reverse. Let $\mathcal{M}_{ij}$ contain all pairs (T, H) consisting of a transplant center T in DSA i and a donor hospital H in DSA j, where the distance between these is $\hat{\delta}_{T,H}$. In 2010, each transplant center T performed t_T transplants where the total number of transplants in DSA i is t_i, and each donor hospital H produced d_H donors where the total number of donors in DSA j is d_j. Then, we calculate the symmetric transplant-volume-weighted distance δ_{ij} as follows:

$$\delta_{ij} = \sum_{(T,H) \in \mathcal{M}_{ij}} \left(\frac{t_T}{t_i}\right)\left(\frac{d_H}{d_j}\right)\hat{\delta}_{TH} + \sum_{(T,H) \in \mathcal{M}_{ji}} \left(\frac{t_T}{t_j}\right)\left(\frac{d_H}{d_i}\right)\hat{\delta}_{TH}. \quad (1)$$

Finally, α_{ijk} is an indicator, set to 1 if $\delta_{ij} < \delta_{ik}$, and 0 if not. The α_{ijk} appear in the constraints, which enforce that every DSA i must be assigned to the district with the nearest center.

To calculate the symmetric transplant-volume-weighted transport time τ_{ij} between DSAs i and j, the transport times are weighted similarly to Equation (1), replacing distance $\hat{\delta}_{TH}$ with estimated transport time $\hat{\tau}_{TH}$ between transplant center T and donor hospital H. Gentry et al. (2014) report our methodology for estimating the transport time between any two hospitals. Let $\bar{\tau}$ be the upper bound on estimated transport time between any DSA and its district's center.

Then, the liver redistricting problem minimizes the sum of absolute differences between the ideal number of donors and the actual number of donors in each district:

$$\text{Minimize:} \quad \sum_{k \in \mathcal{K}} \left| \sum_{i \in \mathcal{I}} p_i W_{ik} - \sum_{i \in \mathcal{I}} d_i W_{ik} \right| \quad (2)$$

$$\text{subject to:} \quad \sum_{k \in \mathcal{K}} W_{ik} = 1 \quad \text{for all } i \in \mathcal{I} \quad (3)$$

$$W_{ik} - Y_k \leq 0 \quad \text{for all } i \in \mathcal{I} \text{ and } k \in \mathcal{K} \quad (4)$$

$$\sum_{k \in \mathcal{K}} Y_k = N \quad (5)$$

$$W_{ik}\tau_{ik} \leq \bar{\tau} \quad \text{for all } i \in \mathcal{I} \text{ and } k \in \mathcal{K} \tag{6}$$

$$\sum_{i \in \mathcal{I}} h_i W_{ik} \geq (\underline{h})Y_k \quad \text{for all } k \in \mathcal{K} \tag{7}$$

$$\sum_{k \in \mathcal{K}} \alpha_{ijk} W_{ik} \leq 1 - Y_j \quad \text{for all } i \in \mathcal{I} \text{ and } j \in \mathcal{K} \tag{8}$$

$$W_{ik} \in \{0, 1\} \quad \text{for all } i \in \mathcal{I} \text{ and } k \in \mathcal{K} \tag{9}$$

$$Y_k \in \{0, 1\} \quad \text{for all } k \in \mathcal{K}. \tag{10}$$

Constraint (3) guarantees that every DSA is assigned to exactly one district. Constraint (4) ensures that Y_k is 1 whenever W_{ik} is 1 for some i, that is, when any DSA is assigned to a district with center at DSA k. The number of districts to be designed is set in Constraint (5). Constraint (6) sets an upper bound of $\bar{\tau}$ on the transplant-volume-weighted time to transport the kidney from the center of any district to any DSA in that district. We had to exclude the two DSAs located in Hawaii and in Puerto Rico from this constraint, or else the problem would have been infeasible for all parameter settings. Constraint (7) sets the minimum number of transplant centers in any district at $\underline{h}$. Finally, Constraint (8) guarantees that every DSA i is assigned to the district whose center is nearest.

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